

H

Mrs. Prior

Pre-Calc Honors, P2

2 December 2013

### The Amusement Park Writing Assignment

Dear Mr. Gatsby,

It has come to our attention that you are interested in knowing if additional security is necessary. We understand that your current security force is sufficient to handle up to 5500 people. Any periods of times when there are more than 5500 people in the park demand additional security. This is a deep concern because we understand that you have high standards for your businesses and you strive to ensure absolute safety. Thus, we urge you to consider both the total number of people who have entered the park and the total number of people who have left the park. If you take both into account, you will be able to create a function determining how many people are in the park at a given time, which you may use to calculate the specific times during which additional security is essential.

We recommend that there is an increased amount of security between 10:53:24 am and 5:11:24 pm. We figured this out by finding the equations to model the number of people at any

time  $t$  (in hours) since the park opening time. This

equation  $y = \begin{cases} 900\sqrt{12t} + 200 & 0 \leq t < 4 \\ 900\sqrt{12t} + 200 - 800(t - 4) & 4 \leq t \leq 16 \end{cases}$  is:

$N(t) =$

To derive this function, we first defined our variables:

$N(t) = E(t) - L(t) \Rightarrow$  given general equation form

$t =$  time (in hours)

$N(t) =$  total # people in the park at a given time

Hilary Wong 12/1/13 11:39 AM

**Comment:** <https://drive.google.com/a/students.shrewsbury.k12.ma.us/folderview?id=0B1Es2eqM8N20andOTDM1VUk0ZFE&usp=sharing#>

Check List:  
and explain in detail how and why you came up with your equation  
why you are making your recommendation.  
Label and refer to graphs/tables(Ex. See diagram A or See Graph 1).  
check rubric: communication and critical thinking rubrics.  
\*\*\*\*\*CHECK WEBSITE, LAST PAGE FOR INSTRUCTIONS

Hilary Wong 12/1/13 12:18 PM

**Comment:** Make sure you explain the problem to the reader.

- Indicate to the reader why the problem is interesting or important.
- Give some indication of what will follow in the body paragraph(s).

Hilary Wong 12/1/13 12:18 PM

**Comment:** Body paragraphs (from rubric):

Explain your process and choices in a logical order. Use transitional words and phrases.

- Be sure to keep your audience in mind... you don't want the reader to have to ask "Why?" but you

also don't want to overwhelm the reader with details. Thinking about the reader will help you to

decide which details you need to include and which you should leave out.

- State any physical and mathematical assumptions that you made.

- Be sure any variable used is clearly defined (e.g. if you use the equation  $y = mx + b$ , state what  $m$

and  $b$  represents) and use math symbols and vocabulary where appropriate (e.g. instead of saying

"x plus two equals seven," write " $x+2=7$ "; instead of saying "the curved line goes up," say "the

curve increases")



$E(t)$  = total # people who have **entered** park at  $t$

$L(t)$  = total # people who have **left** park at  $t$

Secondly, we found several equations to model the information we have about the patterns of people entering and leaving.

The first piece of the final equation we found was  $L(t) = 800(t - 4)$ . This equation describes the total number of people who leave after a given number of hours. Since approximately 800 people LEAVE the park PER hour, starting at 12:00pm, there is a constant rate of change in the number of people who leave PER hour. Thus, a linear function with the slope of 800 can model this pattern of change. The horizontal translation to the right accounts for the fact that 800 people start leaving AFTER 4 hours from opening time.

The next equation we found was  $E(t) = 900\sqrt{12t+200}$ . This equation describes the total number of people who enter after a given number of hours. First, we determined from the data that a square root function best models the data. Given the values of Table 1, we calculated the differences between the y-values of the three different points. The difference in y-values of the points (0, 200) and (6, 7836) is  $7836 - 200 = 7636$ . The difference in y-values of the points (6, 7836) and (12, 11000) is  $11000 - 7836 = 3164$ . As x-values increases by a constant rate (increment of 6 hours), y-values increase at a decreasing rate. One of the characteristics of the square root function family is that the pattern of change increases at a decreasing rate as x increases at a constant rate. An exponential function would not model this data because the function does not exhibit asymptotic behavior and approach one end value.

The final function  $N(t)$  we found was:

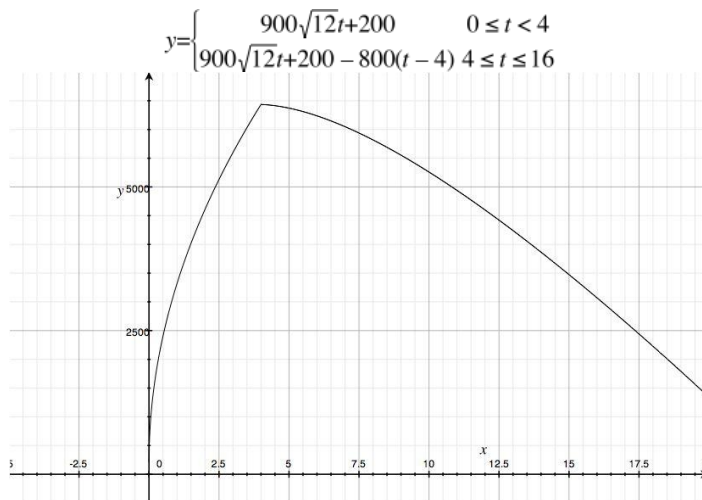
$$N(t) = \begin{cases} 900\sqrt{12t+200} & 0 \leq t < 4 \\ 900\sqrt{12t+200} - 800(t - 4) & 4 \leq t \leq 16 \end{cases}$$

This final equation, which combined the previous two pieces, was BASED ON, but not an exact

substitution, of the general form of  $N(t) = E(t) - L(t)$ . This is the equation that describes the total number of people in the park at a given time. The final equation for  $N(t)$  should NOT be strictly based on the substitution of  $E(t)$  and  $L(t)$  into this general “rule” for calculating the total number of people at a given time because it would be incomplete:  $N(t) = 900\sqrt{12t+200} - 800(t - 4)$ . This equation, which includes  $L(t)$ , only works starting at the 4th hour because people actually start leaving the park at 12:00pm, 4 hours after opening time. There is a restricted domain of:  $4 \leq t \leq 16$  because the time when this starts to happen is greater than or equal to 4 hours ( $t \geq 4$ ) and because the time when this stops happening is when the park closes at midnight (16 hours after opening). To account for the 4 earliest hours during where NO ONE leaves but only ENTERS, we must include this piece of the function  $E(t)$ :  $900\sqrt{12t+200}$ . The restricted domain of this piece of the function  $N(t)$  is:  $0 \leq t < 4$ . There is the inequality “ $<$ ” (and not  $\leq$ ) because this piece of the function works for all values UP TO but NOT INCLUDING 12:00 pm (the 4th hour). Due to the restricted domains, this is a piecewise function.

[See Graph 1 for a display of this function.]

**Graph 1.**



Now that we have our function, we can use it to calculate the exact times when additional security is needed. Operators will need to hire additional security for time periods during which the total number of people in the park at those times is greater than 5500 people. According to Graph 2, there should be 2 points of intersection. These points of intersection represent the start and end times of when security should be added.

**Graph**

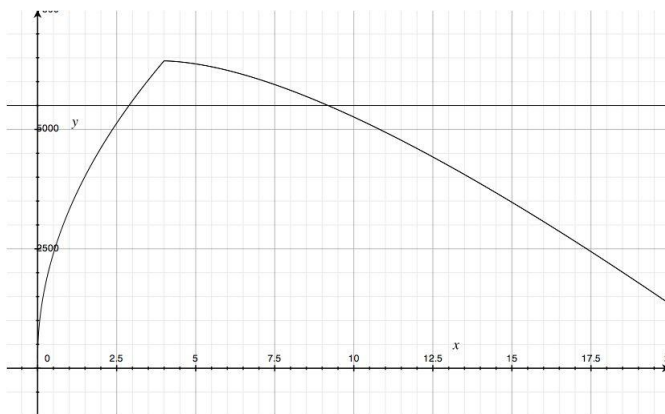
**2:**

$$N(t) = y = \begin{cases} 900\sqrt{12t+200} & 0 \leq t < 4 \\ 900\sqrt{12t+200} - 800(t-4) & 4 \leq t \leq 16 \end{cases}$$

$$Y_4 = 5500$$

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$N(t)$



Because the current security force can only handle 5500 people, we must solve the following equation to determine when more security is necessary:

Hilary Wong 12/5/13 4:20 PM

**Comment:** Explain your process and choices in a logical order. Use transitional words and phrases.

- Be sure to keep your audience in mind... you don't want the reader to have to ask "Why?" but you

also don't want to overwhelm the reader with details. Thinking about the reader will help you to

decide which details you need to include and which you should leave out.

- State any physical and mathematical assumptions that you made.

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and  $b$  represents) and use math symbols and vocabulary where appropriate (e.g. instead of saying

" $x$  plus two equals seven," write " $x+2=7$ "; instead of saying "the curved line goes up," say "the

curve increases")

- Include or reference any equations, diagrams, tables, graphs, pictures, etc. Be sure that if you write

your calculations, draw a graph, etc. outside of the paragraph, those calculations, graphs, etc. are

clearly labeled.

$$5500 = 90012t+200$$

$N(t)$  is equal to 5500 so that we can find the time when the total number of people exceeds the security's limit. We used the equation of the first piece,  $N(t) = 90012t+200$ ,  $0 \leq t < 4$ , because the total number of people in the park reached 5600 by the third hour ( $t = 3$ ), and it was only 4609.082 people in the park in the second hour ( $t = 2$ ) [See Table 3]. The number of visitors in the park must have exceeded 5500 people between the second and third hour. Given this time frame, we must use the equation  $90012t+200$  because  $t = 2$  and  $t = 3$  are within its restricted domain of  $0 \leq t < 4$ .

We set the equation of the first piece,  $90012t+200$ , equal to 5500 because we want to find when the 2 graphs intersect. To solve, we entered 2 equations into the Ti-84 Plus Silver Edition calculator and found the point of intersection. The first point of intersection was (2.89, 5500).

To find the second point of intersection, we have to use the equation of the second piece,  $N(t) = 90012t+200] - 800(t - 4)$ ,  $4 \leq t \leq 16$ , because the second point of intersection, according to Graph 2, is long after the fourth hour ( $t = 4$ ). To solve, we entered 2 equations into the calculator and found the point of intersection. This point of intersection was (9.1875, 5500).

Thus, starting from time 2.89 (the 2.89th hour), or approximately 10:53:04 am, additional security must be added. The "2" in the ones place correspond to the second hour since opening time, which is 10:00. The .89 portion of the hour is (.89 hour)(60 minutes/hour) = 53.4 minutes. The ".4 minutes" equals 24 seconds because (.4 seconds)(60 seconds/minutes). Thus, the time is 10:53:24 am. The ending time for additional security is the approximately the 9.19th hour, or approximately 5:11 pm. The ".19 hour" equals 11.4 minutes because (.19 hour)(60 minutes) = 11.4 minutes. The ".4 minutes" equals 24 seconds because (.4 seconds)(60 seconds/minutes). Thus, the ending time is 5:11:24 pm. Although it is mathematically correct to have additional

Hilary Wong 12/5/13 4:23 PM

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and  $b$  represents) and use math symbols and vocabulary where appropriate (e.g. instead of saying

"x plus two equals seven," write " $x+2=7$ "; instead of saying "the curved line goes up," say "the

curve increases")

- Include or reference any equations, diagrams, tables, graphs, pictures, etc. Be sure that if you write

your calculations, draw a graph, etc. outside of the paragraph, those calculations, graphs, etc. are

security between 10:53:24am and 5:11:24pm, it is more realistic to have extra security from 10:53am to 5:12pm because specifying the seconds is not practical. We rounded 10:53:24 am down to 10:53 am because it is better for additional security to come out earlier and be prepared to manage the influx of people. We rounded 5:11:24 pm up to 5:12 pm because it is better for the security to stay a little bit longer after the number of people have decreased, in case an emergency situation arises.

Overall, it is imperative that you hire additional security to further regulate the amusement park during this season. The time period when more security is necessary is from 10:53 am to 5:12 pm. There are points of the day where the total number of people exceeds 5500, therefore it would certainly be in your best interest to hire a small amount more of security since your current security can only handle 5500 people. The main purpose is to keep your park as safe as possible, so please take our recommendation into consideration. Have a fantastic day with Daisy at the amusement park!

Yours truly,

Hilary & Monica

### Tables:

Table 1. Information Given from the Prompt

Time	8:00am	2:00pm	8:00pm
t	0	6	12
E(t)	200	7836	11,000

Table 2. Table of Values Including Cumulative Work

time	t	$E(t) = 900 + 200$	$L(t) = 800(t - 4), t \geq 4$	$N(t) = E(t) - L(t)$
8:00 am	0	900 +200 = 200		200

Hilary Wong 12/1/13 12:19 PM

**Comment:** Make sure you answer the question(s) being asked!

- Reiterate why this problem and solution are interesting or important

Hilary Wong 12/1/13 12:20 PM

**Comment:** • Write in the first person, preferably "we" not "I." This is often the convention in mathematical

writing.

- Don't start a sentence with a formula or variable.
- Try to avoid overuse of pronouns like "it" and "that"...be specific.

- Specific examples can help make your writing more persuasive. You can help a reader understand

an abstract general argument by showing how the argument applies to a specific case.

- You can also use "extreme" cases to show the limits of an argument.

- Make sure that what you write is relevant to the problem. Including extraneous comments or

information demonstrates a lack of understanding and reduces the overall effectiveness of your

mathematical writing. Do NOT try to "hedge your bets" by writing everything you can think of so

you don't lose points- you will lose points on clarity!

- Write as simply and directly as possible. No one likes to read "ponderous, pretentious prose."



<b>9:00 am</b>	<b>1</b>	900 +200 = 3317.691		3317.691
<b>10:00 am</b>	<b>2</b>	900 +200 = 4609.082		4609.082
<b>11:00 am</b>	<b>3</b>	900 +200 = 5600		5600
<b>12:00pm</b>	<b>4</b>	900 +200 = 6435.383	$800(4 - 4) =$ $800(0) = 0$	$6435.383 - 0 =$ $6435.383$
<b>1:00 pm</b>	<b>5</b>	900 +200 = 7171.370	$800(5 - 4) =$ $800(1) = 800$	$7171.37 - 800 =$ $6371.37$
<b>2:00 pm</b>	<b>6</b>	900 +200 = 7836.753	$800(6 - 4) =$ $800(2) = 1600$	$7836.753 - 1600$ $= 6236.753$
<b>3:00 pm</b>	<b>7</b>	900 +200 = 8448.636	$800(7 - 4) =$ $800(3) = 2400$	$8448.636 - 2400$ $= 6048.636$
<b>4:00 pm</b>	<b>8</b>	900 +200 = 9018.163	$800(8 - 4) =$ $800(4) = 3200$	$9018.163 - 3200$ $= 5818.163$
<b>5:00 pm</b>	<b>9</b>	900 +200 = 9553.074	$800(9 - 4) =$ $800(5) = 4000$	$9553.074 - 4000$ $= 5553.074$
<b>6:00 pm</b>	<b>10</b>	900 +200 = 10059.006	$800(10 - 4) =$ $800(6) = 4800$	$10059.006 -$ $4800 = 5259.006$
<b>7:00 pm</b>	<b>11</b>	900 +200 = 10540.213	$800(11 - 4) =$ $800(7) = 5600$	$10540.213 -$ $5600 = 4940.213$
<b>8:00 pm</b>	<b>12</b>	900 +200 = 11000	$800(12 - 4) =$ $800(8) = 6400$	$11000 - 6400 =$ $4600$
<b>9:00 pm</b>	<b>13</b>	900 +200 = 11440.996	$800(13 - 4) =$ $800(9) = 7200$	$11440.996 -$ $7200 = 4240.996$
<b>10:00 pm</b>	<b>14</b>	900 +200 = 11865.333	$800(14 - 4) =$ $800(10) = 8000$	$11865.333 -$ $8000 = 3865.333$
<b>11:00 pm</b>	<b>15</b>	900 +200 = 12274.767	$800(15 - 4) =$ $800(11) = 8800$	$12274.767 -$ $8800 = 3474.767$
<b>12:00 am</b>	<b>16</b>	900 +200 = 12670.766	$800(16 - 4) =$ $800(12) = 9600$	$12670.766 -$ $9600 = 3070.766$

Table 3. Simplified Table 2 of Values



time	t	E(t)
8:00 am	0	200
9:00 am	1	3317.691
10:00 am	2	4609.082
11:00 am	3	5600
12:00pm	4	6435.383
1:00 pm	5	6371.37
2:00 pm	6	6236.753
3:00 pm	7	6048.636
4:00 pm	8	5818.163
5:00 pm	9	5553.074
6:00 pm	10	5259.006
7:00 pm	11	4940.213
8:00 pm	12	4600
9:00 pm	13	4240.996
10:00 pm	14	3865.333
11:00 pm	15	3474.767
12:00 am	16	3070.766